are very powerful and efficient means of approximating numerically the solutions of an enormous class of initial value problems. Specific implementations of one or more of them are widely available in commercial software packages.

PROBLEMS

In each of Problems 1 through 6 find approximate values of the solution of the given initial value problem at $t = 0.1, 0.2, 0.3$, and 0.4. Compare the results with those obtained by using other methods and with the exact solution (if available).

(a) Use the Runge-Kutta method with $h = 0.1$.

(b) Use the Runge-Kutta method with $h = 0.05$.

1. $y' = 3 + t - y, \quad y(0) = 1$
2. $y' = 5t - 3\sqrt{y}, \quad y(0) = 2$
3. $y' = 2y - 3t, \quad y(0) = 1$
4. $y' = 2t + e^{-ty}, \quad y(0) = 1$
5. $y' = \dfrac{y^2 + 2ty}{3 + t^2}, \quad y(0) = 0.5$
6. $y' = (t^2 - y^2)\sin y, \quad y(0) = -1$

In each of Problems 7 through 12 find approximate values of the solution of the given initial value problem at $t = 0.5, 1.0, 1.5$, and 2.0. Compare the results with those obtained by other methods.

(a) Use the Runge–Kutta method with $h = 0.1$.

(b) Use the Runge–Kutta method with $h = 0.05$.

7. $y' = 0.5 - t + 2y, \quad y(0) = 1$
8. $y' = 5t - 3\sqrt{y}, \quad y(0) = 2$
9. $y' = \sqrt{t + y}, \quad y(0) = 3$
10. $y' = 2t + e^{-ty}, \quad y(0) = 1$
11. $y' = (4 - ty)/(1 + y^2), \quad y(0) = -2$
12. $y' = (y^2 + 2ty)/(3 + t^2), \quad y(0) = 0.5$
13. Confirm the results in Table 8.3.1 by executing the indicated computations.
14. Consider the initial value problem

$$y' = t^2 + y^2, \qquad y(0) = 1.$$

(a) Draw a direction field for this equation.

(b) Use the Runge–Kutta method or another method to find approximate values of the solution at $t = 0.8, 0.9$, and 0.95. Choose a small enough step size so that you believe your results are accurate to at least four digits.

(c) Try to extend the calculations in part (b) to obtain an accurate approximation to the solution at $t = 1$. If you encounter difficulties in doing this, explain why you think this happens. The direction field in part (a) may be helpful.

15. Consider the initial value problem

$$y' = 3t^2/(3y^2 - 4), \qquad y(0) = 0.$$

(a) Draw a direction field for this equation.

(b) Estimate how far the solution can be extended to the right. Let t_M be the right endpoint of the interval of existence of this solution. What happens at t_M to prevent the solution from continuing farther?

(c) Use the Runge–Kutta method with various step sizes to determine an approximate value of t_M.

See parts (d) and (e) on the next page.