

Math 432 HW 3.4 Solutions

Assigned: 5, 6, 9, and 18.

Selected for Grading: 18

Solutions:

5. This exercise is in the same setting as example 1 of the text.

Starting with a force diagram and using Newton's second law would give the starting model

$$m \frac{dv}{dt} = mg - bv$$

Solving that differential equation as is done in the text would give the formula

$$v(t) = \frac{mg}{b} + \left(v_0 - \frac{mg}{b}\right) e^{-bt/m}$$

And the equation for position would be found similarly:

$$y(t) = \frac{mg}{b} t + \frac{m}{b} \left(v_0 - \frac{mg}{b}\right) (1 - e^{-bt/m})$$

where we assume $y(0) = 0$ and downwards is the positive direction.

From the information given in the problem we have $m = 5$, $b = 10$, and $v_0 = 50$. Also $g = 9.81$.

Substituting these values in the above formula and simplifying give the equation of motion:

$$y(t) = 4.905t + 22.5475(1 - e^{-2t}).$$

To answer the remaining question, transform the question to determining when the object has gone 500 meters. So I'll set $y(t) = 500$ and solve for t .

$$4.905t + 22.5475(1 - e^{-2t}) = 500$$

There are no algebraic tools that I can use to solve this problem, so I'll use a calculator.

My TI-83 gave me $t \approx 97.339959$.

The object would hit the ground in approximately 97.34 seconds.

6. The only difference between this exercise and the previous one is in the values of the constants. In short, our equation of motion is

$$\begin{aligned} y(t) &= \frac{8g}{16} t + \frac{8}{16} \left(-20 - \frac{8g}{16}\right) (1 - e^{-\frac{16t}{8}}) \\ &= \frac{g}{2} t - \frac{1}{2} \left(20 + \frac{g}{2}\right) (1 - e^{-2t}) = 4.905t - 12.4525(1 - e^{-2t}) \end{aligned}$$

The object hits the ground when $y(t) = 100$:

$$4.905t - 12.4525(1 - e^{-2t}) = 100$$

$$t \approx 22.926096$$

The object hits the ground in approximately 22.93 seconds.

9. In this exercise's context there are three forces acting on the object: one due to gravity, one due to water resistance, and one due to buoyancy. So we have to come up with a new differential equation. Starting with a force diagram (which I'll include later when I get the chance) and using Newton's second law, we get

$$m \frac{dv}{dt} = mg - \frac{1}{40}mg - 10v$$

which simplifies to

$$\frac{dv}{dt} = \frac{39}{40}g - \frac{10v}{m} = \frac{39g}{40} - \frac{v}{10} = \frac{39g - 4v}{40}$$

which is separable:

$$\int \frac{dv}{39g - 4v} = \int \frac{dt}{40}$$

$$-\frac{1}{4} \ln|39g - 4v| = \frac{t}{40} + C_1$$

$$\ln|39g - 4v| = -t/10 + C_2$$

$$39g - 4v = C_3 e^{-t/10}$$

$$v = 39g/4 + C_4 e^{-t/10}$$

Since $v(0) = 0$, then $v(t) = (39g/4)(1 - e^{-t/10})$.

Integrating this equation gives $y(t) = (39g/4)(t + 10e^{-t/10} + C)$.

Using the initial condition $y(0) = 0$ gives that forces $C = -10$.

So for our equation of motion we get

$$y(t) = (39g/4)(t + 10e^{-t/10} - 10).$$

Setting the formula for velocity equal to 70 and solving for t :

$$[(39 \times 9.81)/4](1 - e^{-t/10}) = 70$$

$$1 - e^{-t/10} = (70 \times 4)/(39 \times 9.81)$$

$$e^{-t/10} = 1 - (70 \times 4)/(39 \times 9.81)$$

$$-t/10 = \ln[1 - (70 \times 4)/(39 \times 9.81)]$$

$$t = -10 \ln[1 - (70 \times 4)/(39 \times 9.81)]$$

$$t \approx 13.16223458$$

The object will reach a velocity of 70 m/sec in about 13.16 seconds.

18. In this setting we have two pertinent forces acting on the object, that contributed by gravity, and that contributed by friction. The force due to gravity contributes $mg \sin(30^\circ) = mg/2$ and the normal forces contributes $-\mu N = -0.2(mg \cos 30^\circ) = -0.2 \cdot \frac{\sqrt{3}}{2} mg$. From these parts we get the differential equation

$$m \frac{dv}{dt} = \frac{1}{2} mg - 0.2 \cdot \frac{\sqrt{3}}{2} mg$$

$$\frac{dv}{dt} = \frac{1}{2} g - 0.2 \cdot \frac{\sqrt{3}}{2} g = \left(\frac{1}{2} - 0.2 \cdot \frac{\sqrt{3}}{2} \right) g = \frac{1 - 0.2\sqrt{3}}{2} g = g \left(\frac{5 - \sqrt{3}}{10} \right)$$

$$v(t) = \left(\frac{5 - \sqrt{3}}{10} \right) gt + C$$

$v(0) = 0$, so $C = 0$ and we have

$$v(t) = \left(\frac{5 - \sqrt{3}}{10} \right) gt$$

Integrating both sides of this equation with respect to t gives

$$x(t) = \left(\frac{5 - \sqrt{3}}{20} \right) gt^2 + C$$

$x(0) = 0$, so $C = 0$.

The equation of motion is

$$x(t) = \left(\frac{5 - \sqrt{3}}{20} \right) gt^2$$

To answer the remaining question I'll have to (1) find the value of t when the object reaches the bottom and then (2) use that value as input for the velocity function.

$$\left(\frac{5 - \sqrt{3}}{20} \right) gt^2 = 5$$

$$t^2 = \frac{100}{(5 - \sqrt{3})g}$$

$$t = \sqrt{\frac{100}{(5 - \sqrt{3})g}}$$

(You can ignore the negative value for t .)

$$t \approx 1.766151068$$

$$v \left(\sqrt{\frac{100}{(5 - \sqrt{3})g}} \right) = \left(\frac{5 - \sqrt{3}}{10} \right) g \left(\sqrt{\frac{100}{(5 - \sqrt{3})g}} \right) \approx 5.662029811$$

The velocity of the object when it hits the bottom is approximately 5.66 m/sec.