Math 432 HW 4.3 Solutions

<u>Assigned:</u> 1, 5, 8, 11, 17, 20, 21, 24, 27, 32, and 33 (a & b)

<u>Selected for Grading:</u> 5, 8, 20, 33

<u>Solutions:</u>

1. $y'' + 9y = 0$
   $r^2 + 9 = 0$
   $r^2 = -9$
   $r = \pm 3i$. (So $\alpha = 0$ and $\beta = 3$.)

   General solution: $c_1 \cos 3t + c_2 \sin 3t$

5. $w'' + 4w' + 6w = 0$
   $r^2 + 4r + 6 = 0$
   $r = -2 \pm \sqrt{2}i$ (So $\alpha = -2$ and $\beta = \sqrt{2}$.

   General solution: $e^{-2t}\left(c_1 \cos \sqrt{2}t + c_2 \sin \sqrt{2}t\right)$

8. $4y'' + 4y' + 6y = 0$
   $4r^2 + 4r + 6 = 0$
   $2r^2 + 2r + 3 = 0$
   $r = -\frac{1}{2} \pm \frac{\sqrt{5}}{2} i$

   General solution: $y = e^{-t/2}\left(c_1 \cos \frac{\sqrt{5}}{2} t + c_2 \sin \frac{\sqrt{5}}{2} t\right)$.

11. $z'' + 10z' + 25z = 0$
   $r^2 + 10r + 25 = 0$
   $(r + 5)^2 = 0$

   General solution: $z = c_1 e^{-5t} + c_2 t e^{-5t}$

17. $y'' - y' + 7y = 0$
   $r^2 - r + 7 = 0$
   $r = \frac{1}{2} \pm \frac{3\sqrt{3}}{2} i$

   General solution: $y = e^{t/2}\left(c_1 \cos \frac{3\sqrt{3}}{2} t + c_2 \sin \frac{3\sqrt{3}}{2} t\right)$

20. $y''' - y'' + 2y = 0$
   $r^3 - r^2 + 2 = 0$
   Note that $r = -1$ is a root, so $(r + 1)$ is a factor: $r^3 - r^2 + 2 = (r + 1)(r^2 - 2r + 2)$.
   The roots are $r_1 = -1$, $r_2 = 1 + i$, $r_3 = 1 - i$.

   General solution: $y = c_1 e^{-t} + e^{t}(c_2 \cos t + c_3 \sin t)$

21. $y'' + 2y' + 2y = 0$; $y(0) = 2$, $y'(0) = 1$.
$r^2 + 2r + 2 = 0$
$r = -1 \pm i$

$y = e^{-t}(c_1 \cos t + c_2 \sin t)$
$y' = e^{-t}(-c_1 \sin t + c_2 \cos t) - e^{-t}(c_1 \cos t + c_2 \sin t)$

From the initial conditions we get
$c_1 = 2$ and
$c_2 - c_1 = 1$, and hence $c_2 = 3$.

Solution: $y = e^{-t}(2 \cos t + 3 \sin t)$

24. $y'' + 9y = 0$; $y(0) = 1$, $y'(0) = 1$.
$r^2 + 9 = 0$
$r = \pm 3i$

$y = c_1 \cos 3t + c_2 \sin 3t$
$y' = -3c_1 \sin 3t + 3c_2 \cos 3t$

Initial conditions give
$c_1 = 1$
$3c_2 = 1$. $c_2 = 1/3$

Solution: $y = \cos 3t + (1/3) \sin 3t$

27. $y''' - 4y'' + 7y' - 6y = 0$; $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$
$r^3 - 4r^2 + 7r - 6 = 0$. From a graph of this function I guessed and verified that $r = 2$ is a root. So $r - 2$ is a factor: $r^3 - 4r^2 + 7r - 6 = (r - 2)(r^2 - 2r + 3)$.
Roots: $r = 2, 1 + \sqrt{2}i, 1 - \sqrt{2}i$.

$y = c_1 e^{2t} + e^t\left(c_2 \cos \sqrt{2}t + c_3 \sin \sqrt{2}t\right)$
$y' = 2c_1 e^{2t} + e^t\left(\left(c_2 + \sqrt{2}c_3\right) \cos \sqrt{2}t + \left(c_3 - \sqrt{2}c_2\right) \sin \sqrt{2}t\right)$
$y'' = 4c_1 e^{2t} + e^t\left[\left(2\sqrt{2}c_3 - c_2\right) \cos \sqrt{2}t - \left(2\sqrt{2}c_2 + c_3\right) \sin \sqrt{2}t\right]$

Initial conditions give the system of equations
$c_1 + c_2 = 1$
$2c_1 + c_2 + \sqrt{2}c_3 = 0$
$4c_1 - c_2 + 2\sqrt{2}c_3 = 0$
whose solution is $c_1 = 1$, $c_2 = 0$, $c_3 = -\sqrt{2}$.

Solution: $y = e^{2t} - \sqrt{2}e^t \sin \sqrt{2}t$

32. (a) We start with $my''(t) + by'(t) + ky(t) = 0$ and are given that $b = 0$, $m = 10$, and $k = 250$.

$10y'' + 250y = 0$; $y(0) = 0.3$, $y'(0) = -0.1$

$10r^2 + 250 = 0$

$r^2 = -25$

$r = \pm 5i$

$y = c_1 \cos 5t + c_2 \sin 5t$

$y' = -5c_1 \sin 5t + 5c_2 \cos 5t$

$c_1 = 0.3$

$5c_2 = -0.1$, so $c_2 = -0.02$

Solution: $y = 0.3 \cos 5t - 0.02 \sin 5t$

(b) The frequency is $5/(2\pi)$.

33. (a) With all the givens we have the IVP $10y'' + 60y' + 250y = 0$; $y(0) = 0.3$, $y'(0) = -0.1$.

$10r^2 + 60r + 250 = 0$

$r^2 + 6r + 25 = 0$

$r = -3 \pm 4i$

$y = e^{-3t}[c_1 \cos 4t + c_2 \sin 4t]$

$y' = e^{-3t}[(-3c_1 + 4c_2) \cos 4t + (-3c_2 - 4c_1) \sin 4t]$

$c_1 = 0.3$

$-3c_1 + 4c_2 = -0.1$, $4c_2 = -0.1 + 0.9 = 0.8$, $c_2 = 0.2$

Solution: $y = e^{-3t}(0.3 \cos 4t + 0.2 \sin 4t)$.

(b) The frequency of oscillation is $2/\pi$.