Math 432 – HW 4.4 Solutions

Assigned: 1, 3, 5, 6, 9, 11, 13, 15, 16, 21, 27, and 28

Selected for Grading:

Solutions.

1. The method of undetermined coefficients can not be applied to find a particular solution to the differential equation $y'' + 2y' - y = t^{-1}e^{t}$. {The power of t is not a nonnegative integer.}

3. The method of undetermined coefficients can be applied to find a particular solution to the differential equation $x'' + 5x' - 3x = 3^t$. {For a justification, notice that 3^t is equal to $e^{(\ln 3)t}$.}

5. The method of undetermined coefficients can not be applied to find a particular solution to the differential equation $y''(\theta) + 3y'(\theta) - y(\theta) = \sec\theta$. {The method does not work with all trigonometric functions, only sine and cosine.}

6. The method of undetermined coefficients can be applied to find a particular solution to the differential equation $2\omega''(x) - 3\omega(x) = 4x\sin^2 x + 4x\cos^2 x$. {Because $4x\sin^2 x + 4x\cos^2 x$ simplifies to $4x$.}

9. $y'' + 3y = -9$
   $r^2 + 3 = 0$
   $r = \pm\sqrt{3}i$
   $y_h = c_1\cos\sqrt{3}t + c_2\sin\sqrt{3}t$

   Let $y_p = A$.
   Then $y_p' = y_p'' = 0$.
   Substituting gives
   $0 + 3A = -9$
   $A = -3$

   $y_p \equiv -3$. {It's nice to start off with an easy one.}

11. $2z'' + z = 9e^{2t}$
   $2r^2 + 1 = 0$
   $r = \pm\sqrt{1/2}i$
   $y_h = c_1\cos\sqrt{1/2}t + c_2\sin\sqrt{1/2}t$

   Let $y_p = Ae^{2t}$.
   Then $y_p' = 2Ae^{2t}$ and $y_p'' = 4Ae^{2t}$.
   Substituting gives
   $e^{2t}[2\cdot 4A + A] = 9e^{2t}$
   $9A = 9$
   $A = 1$

   $y_p = e^{2t}$.

13. $y'' - y' + 9y = 3 \sin 3t$

$r^2 - r + 9 = 0$

$$r = \frac{1}{2} \pm \frac{\sqrt{35}}{2} i$$

$$y_h = e^{t/2}\left[c_1 \cos \sqrt{35}t + c_2 \sin \sqrt{35}t\right]$$

Let $y_p = A \cos 3t + B \sin 3t$.

Then $y_p' = -3A \sin 3t + 3B \cos 3t$.

And $y_p'' = -9A \cos 3t - 9A \sin 3t$.

Substituting gives

$-9A \cos 3t - 9A \sin 3t - (-3A \sin 3t + 3B \cos 3t) + 9(A \cos 3t + B \sin 3t)$

$= (-9A - 3B + 9A)(\cos 3t) + (-9A + 3A + 9A)(\sin 3t)$

$= -3B \cos 3t + 3A \sin 3t$

$= 3 \sin 3t$

So $B = 0$ and $A = 1$.

$y_p = \cos 3t$

15. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^x$

$r^2 - 5r + 6 = 0$

$(r - 6)(r + 1) = 0$

$y_h = c_1 e^{-x} + c_2 e^{6x}$

$y_p = (Ax + B)e^x$

$y_p' = (Ax + A + B)e^x$

$y_p'' = (Ax + 2A + B)e^x$

Substituting:

$(Ax + 2A + B)e^x - 5[(Ax + A + B)e^x] + 6[(Ax + B)e^x]$

$= e^x[Ax - 5Ax + 6Ax + 2A + B - 5A - 5B + 6B]$

$= e^x[2Ax + (-3A + 2B)]$

$= xe^x$

So

$2A = 1$ and $-3A + 2B = 0$

$A = 1/2$ and $B = 3/4$.

$y_p = xe^x/2 + 3e^x/4$

16. $\theta''(t) - \theta(t) = t \sin t$

$r^2 - 1 = 0$

$r = \pm 1$

$y_h = c_1 e^{-t} + c_2 e^{t}$

$\theta_p = (At + B)\cos t + (Ct + D)\sin t$

$\theta_p' = (At + B)(-\sin t) + A\cos t + (Ct + D)\cos t + C\sin t$

$= (Ct + A + D)\cos t + (-At - B + C)\sin t$

$\theta_p'' = (Ct + A + D)(-\sin t) + C\cos t + (-At - B + C)\cos t - A\sin t$

$= (-At - B + 2C)\cos t + (-Ct - 2A - D)\sin t$

Substituting:

$(-At - B + 2C)\cos t + (-Ct - 2A - D)\sin t - [(At + B)\cos t + (Ct + D)\sin t]$

$= (-At - B + 2C - At - B)\cos t + (-Ct - 2A - D - Ct - D)\sin t$

$= (-2At - 2B + 2C)\cos t + (-2Ct - 2A - 2D)\sin t$

$= -2At\cos t + (-2B + 2C)\cos t - 2Ct\sin t + (-2A - 2D)\sin t$

$= t\sin t$

So

$-2A = 0$

$-2B + 2C = 0$

$-2C = 1$

$-2A - 2D = 0$

These give

$A = 0$

$B = C$

$C = -1/2$

$D = A$ So $A = 0$, $B = -1/2$, $C = -1/2$, and $D = 0$.

$\theta_p = (-1/2)\cos t + (-t/2)\sin t = -(\cos t + t\sin t)/2$

21. $x''(t) - 4x'(t) + 4x(t) = te^{2t}$

$r^2 - 4r + 4 = 0$

$(r - 2)^2 = 0$

$x_h = c_1 e^{2t} + c_2 t e^{2t}$

$x_p = t^2(At + B)\, e^{2t} = e^{2t}(At^3 + Bt^2)$

$x_p' = e^{2t}(3At^2 + 2Bt) + 2(At^3 + Bt^2)e^{2t}$

$= e^{2t}(2At^3 + (3A + 2B)t^2 + 2Bt)$

$x_p'' = e^{2t}\,(6At^2 + 2(3A + 2B)t + 2B) + 2(2At^3 + (3A + 2B)t^2 + 2Bt)e^{2t}$

$= e^{2t}(4At^3 + (6A + 2(3A + 2B))t^2 + (2(3A + 2B) + 2B)t + 2B)$

$= e^{2t}(4At^3 + (12A + 4B)t^2 + (6A + 8B)t + 2B)$

Substituting:

$e^{2t}[(4At^3 + (12A + 4B)t^2 + (6A + 8B)t + 2B) - 4(2At^3 + (3A + 2B)t^2 + 2Bt) + 4(At^3 + Bt^2)]$

$= e^{2t}[(4A - 8A + 4A)t^3 + (12A + 4B - 12A - 8B + 4B)t^2 + (6A + 8B - 8B)t + 2B]$

$= e^{2t}[0t^3 + 0t^2 + 6At + 2B]$

$= 6Ate^{2t} + 2Be^{2t}$

$= te^{2t}$

So

$6A = 1$ and $4B = 0$

$A = 1/6$, $B = 0$.

$x_p = t^3 e^{2t}/6$

27. $y'' + 9y = 4t^3 \sin 3t$

$r^2 + 9 = 0$

$r = \pm 3i$

$y_h = c_1 \cos 3t + c_2 \sin 3t$

$y_p = t\,(A_3t^3 + A_2t^2 + A_1t + A_0)\cos 3t + t\,(B_3t^3 + B_2t^2 + B_1t + B_0)\sin 3t$

28. $y'' + 3y' - 7y = t^4e^t$

$r^2 + 3r - 7 = 0$

$$r = \frac{7 \pm \sqrt{37}}{2}$$

$y_p = (A_4t^4 + A_3t^3 + A_2t^2 + A_1t + A_0)\,e^t$