Math 432 – HW 6.2 Solutions

Assigned: 3, 9, 12, 20

Selected for grading: 20

Solutions.

3. Solve the equation $6z''' + 7z'' - z' - 2z = 0$.
$6r^3 + 7r^2 - r - 2 = 0$
By inspection, $r = -1$ is a zero of this auxiliary equation. So $r + 1$ is a factor of $6r^3 + 7r^2 - r - 2$.
Long division shows that
$6r^3 + 7r^2 - r - 2 = (r + 1)(6r^2 + r - 2)$
So the auxiliary equation is now
$(r + 1)(6r^2 + r - 2) = 0$
$(r + 1)(2r - 1)(3r + 2) = 0$
$r = -1, 1/2, -2/3$.

General solution: $y_h = c_1e^{-x} + c_2e^{x/2} + c_3e^{-2x/3}$.

9. Solve the equation $u''' - 9u'' + 27u' - 27u = 0$.
$r^3 - 9r^2 + 27r - 27 = 0$
By inspection, $r = 3$ is a root.
Long division gives $r^3 - 9r^2 + 27r - 27 = (r - 3)(r^2 - 6r + 9) = (r - 3)(r - 3)(r - 3) = (r - 3)^3$.
So the auxiliary equation has one repeated root: $r_1 = r_2 = r_3 = 3$.

General solution: $y_h = c_1e^{3x} + c_2xe^{3x} + c_3x^2e^{3x}$.

12. $y''' + 5y'' + 3y' - 9y = 0$
$r^3 + 5r^2 + 3r - 9 = 0$
By inspection, $r = 1$ is a root.
Long division gives $r^3 + 5r^2 + 3r - 9 = (r - 1)(r^2 + 6r + 9) = (r - 1)(r + 3)^2$.
So the auxiliary equation has roots $r_1 = 1$, $r_2 = -3$, and $r_3 = -3$.

General solution: $y_h = c_1e^{x} + c_2e^{-3x} + c_3xe^{-3x}$.

20. $y''' + 7y'' + 14y' + 8y = 0$; $y(0) = 1$, $y'(0) = -3$, $y''(0) = 13$.

$r^3 + 7r^2 + 14r + 8 = 0$

By inspection, $r = -2$ is a root.

Long division gives $r^3 + 7r^2 + 14r + 8 = (r + 2)(r^2 + 5r + 4) = (r + 2)(r + 1)(r + 4)$.

So the auxiliary equation has three roots: $r_1 = -1$, $r_2 = -2$, and $r_3 = -4$.

The general solution to the differential equation is $y = c_1e^{-t} + c_2e^{-2t} + c_3e^{-4t}$.

$y(0) = c_1 + c_2 + c_3 = 1$

$y' = -c_1e^{-t} - 2c_2e^{-2t} - 4c_3e^{-4t}$

$y'(0) = -c_1 - 2c_2 - 4c_3 = -3$

$c_1 + 2c_2 + 4c_3 = 3$

$y'' = c_1e^{-t} + 4c_2e^{-2t} + 16e^{-4t}$

$y''(0) = c_1 + 4c_2 + 16c_3 = 13$

This yields the system of equations

$c_1 + c_2 + c_3 = 1$

$c_1 + 2c_2 + 4c_3 = 3$

$c_1 + 4c_2 + 16c_3 = 13$

This system has solution $c_1 = 1$, $c_2 = -1$, $c_3 = 1$.

Solution: $y = e^{-t} - e^{-2t} + e^{-4t}$.