Math 432 – HW 7.4 Solutions

Assigned: 1, 3, 7, 9, 10, 11, 13, 15, 19, 21, 24, 25, 28, 30

Selected for grading: 3, 7, 10, 21, 30

Solutions.

~~Close, but OK~~

① $\cancel{F(s)=}\ \dfrac{6}{(s-1)^4} = \dfrac{3!}{(s-1)^{3+1}}$

$= F(s-1)$ where $F(s) = \dfrac{3!}{s^{3+1}} = \mathcal{L}\{t^3\}$

So $\mathcal{L}^{-1}\left\{\dfrac{6}{(s-1)^4}\right\} = e^t t^3$

③ $\dfrac{s+1}{s^2+2s+10} = \dfrac{s+1}{(s+1)^2+3^2}$

$= \mathcal{L}^{-1} = e^{-t}\cos 3t$

⑦ $F(s) = \dfrac{2s+16}{s^2+4s+13} = \dfrac{2s+16}{(s+2)^2+3^2} = \dfrac{2(s+2)+\cancel{8}\,12}{(s+2)^2+3^2}$

$= 2\cdot\dfrac{s+2}{(s+2)^2+3^2} + 4\cdot\dfrac{3}{(s+2)^2+3^2}$

$f(t) = 2e^{-2t}\cos 3t + 4e^{-2t}\sin 3t$

(9) $$\frac{3s-15}{2s^2-4s+10} = \frac{3(s-5)}{2(s^2-2s+5)}$$

$$= \frac{3}{2}\cdot\frac{s-5}{(s-1)^2+2^2}$$

$$= \frac{3}{2}\left[\frac{s-1}{(s-1)^2+2^2} - 2\cdot\frac{2}{(s-1)^2+2^2}\right]$$

$$= \frac{3}{2}\left[\mathcal{L}\{e^t\cos 2t\} - 2\mathcal{L}\{e^t\sin 2t\}\right]$$

$$= \mathcal{L}\{\tfrac{3}{2}e^t\cos 2t - \tfrac{3}{\cancel{2}}\cdot\cancel{2}e^t\sin 2t\}$$

So $$\mathcal{L}^{-1}\left\{\frac{3s-15}{2s^2-4s+10}\right\} = \frac{3}{2}e^t\cos 2t - 3e^t\sin 2t$$

(10.) $$\frac{s-1}{2s^2+s+6} = \frac{s-1}{2(s^2+\frac{1}{2}s+3)}$$

$$3-\frac{1}{16} = \frac{48-1}{16} = \frac{47}{16}$$

$$= \frac{1}{2}\cdot\frac{s-1}{s^2+\frac{1}{2}s+\frac{1}{16}+\frac{47}{16}}$$

$$= \frac{1}{2}\cdot\frac{s-1}{(s+\frac{1}{4})^2+\frac{47}{16}}$$

$$= \frac{1}{2}\left[\frac{s+\frac{1}{4}}{(s+\frac{1}{4})^2+\frac{47}{16}} - \frac{5}{4}\cdot\frac{\frac{\sqrt{47}}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{47}}{4}\right)^2}\cdot\frac{4}{\sqrt{47}}\right]$$

$$= \frac{1}{2}\left[\mathcal{L}\{e^{-t/4}\cos\tfrac{\sqrt{47}}{4}t\} - \frac{5}{\sqrt{47}}\mathcal{L}\{e^{-t/4}\sin\tfrac{\sqrt{47}}{4}t\}\right]$$

$$\frac{1}{2}e^{-t/4}\cos\frac{\sqrt{47}}{4}t - \frac{5}{2\sqrt{47}}e^{-t/4}\sin\frac{\sqrt{47}}{4}t$$

(11) $$\frac{S^2-26S-47}{(S-1)(S+2)(S+5)} = \frac{A}{S-1} + \frac{B}{S+2} + \frac{C}{S+5}$$

$$= \frac{A(S+2)(S+5) + B(S-1)(S+5) + C(S-1)(S+2)}{(S-1)(S+2)(S+5)}$$

$$A(S^2+7S+10) + B(S^2+4S-5) + C(S^2+S-2)$$

$$= S^2(A+B+C) + S(7A+4B+C) + (10A-5B-2C)$$

$$= S^2 - 26S - 47$$

$$\begin{cases} A+B+C = 1 \\ 7A+4B+C = -26 \\ 10A-5B-2C = -47 \end{cases} \quad TI\text{-}83 \Rightarrow \begin{array}{l} A = -4 \\ B = -1 \\ C = 6 \end{array}$$

$$= \frac{-4}{S-1} - \frac{\cancel{B}\,1}{S+2} + \frac{6\cancel{C}}{S+5}$$

(13.) $$\frac{-2S^2-3S-2}{S(S+1)^2} = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{(S+1)^2}$$

$$A(S+1)^2 + BS(S+1) + CS = A(S^2+2S+1) + B(S^2+S) + CS$$

$$= S^2(A+B) + S(2A+B+C) + A$$

$A = -2$. $\quad A+B = -2 \quad B = 0$. $\quad 2A+B+C = -3 \quad -4 + C = -3 \quad C = 1$

$$\frac{-2}{S} + \frac{1}{(S+1)^2}$$

(15.) $$\frac{-2s^2+8s-14}{(s+1)(s^2-2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5} \leftarrow = (s-1)^2+2^2$$

$$A(s^2-2s+5) + Bs(s+1) + C(s+1)$$

$$= s^2(A+B) + s(-2A+B+C) + (5A+C)$$

$$\begin{cases} A+B = -2 \\ -2A+B+C = 8 \\ 5A + C = -14 \end{cases} \quad \begin{aligned} A &= -3 \\ B &= 1 \\ C &= 1 \end{aligned}$$

$$-\frac{3}{s+1} + \frac{s+1}{s^2-2s+5} = -\frac{3}{s+1} + \frac{(s-1)+2}{(s-1)^2+2^2}$$

(19.) $$\frac{1}{(s-3)(s^2+2s+2)} = \frac{A}{s-3} + \frac{Bs+C}{s^2+2s+2} \leftarrow = (s+1)^2+1^2$$

$$As^2 + 2As + 2A + Bs^2 - 3Bs + Cs - 3C = 1$$

$$\begin{cases} A+B=0 \\ 2A-3B+C=0 \\ 2A-3C=1 \end{cases} \quad \begin{aligned} B &= -A \\ -3C &= 1-2A \\ C &= \frac{2A-1}{3} \end{aligned}$$

$$2A+3A+\frac{2A-1}{3} = 0$$

$$6A+9A+2A = 1$$

$$17A = 1$$

$$A = \frac{1}{17}, \quad B = -\frac{1}{17}, \quad C = \frac{-15}{51} = \frac{-5}{17}$$

$$\frac{1}{17}\left[\frac{1}{s-3} - \frac{s+5}{(s+1)^2+1^2}\right] \leftarrow \text{Good enough.}$$

$$= \frac{1}{17}\left[\frac{1}{s-3} - \frac{s+1}{(s+1)^2+1^2} - 4\cdot\frac{1}{(s+1)^2+1^2}\right]$$

(21) $$F(s) = \frac{6s^2 - 13s + 2}{s(s-1)(s-6)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-6}$$

$$A(s^2 - 7s + 6) + B(s^2 - 6s) + C(s^2 - s)$$

$$= s^2(A + B + C) + s(-7A - 6B - C) + (6A)$$

$$= 6s^2 - 13s + 2$$

$$6A = 2$$

$$A = \frac{1}{3}$$

$$\begin{array}{r} A + B + C = 6 \\ -7A - 6B - C = -13 \\ \hline -6A - 5B \quad = -7 \end{array}$$

$$-5B = -7 + 6A = -7 + 2 = -5$$

$$B = 1$$

$$A + B + C = 6$$

$$C = 6 - A - B$$

$$= 6 - \frac{1}{3} - 1 = \frac{14}{3} = C$$

$$F(s) = \frac{1}{3} \cdot \frac{1}{s} + \frac{1}{s-1} + \frac{14}{3} \cdot \frac{1}{s-6}$$

$$f(t) = \cancel{\tfrac{1}{3}e} = \frac{1}{3} + e^{t} + \frac{14}{3}e^{6t}$$

(24) $F(s) = \dfrac{7s^2 - 41s + 84}{(s-1)(s^2-4s+13)} = \dfrac{A}{s-1} + \dfrac{Bs+C}{(s-2)^2+3^2}$

$$As^2 - 4As + 13A + Bs^2 - Bs + Cs - C = 7s^2 - 41s + 84$$

$$s^2(A+B) + s(-4A - B + C) + (13A - C) = 7s^2 - 41s + 84$$

$$\begin{aligned} A + B &= 7 \\ -4A - B + C &= -41 \\ 13A - C &= 84 \end{aligned} \qquad \cancel{A = 7 - B} \qquad \cancel{A = \frac{C+84}{13}}$$

$$B = 7 - A$$
$$C = 13A - 84$$

$$-4A - B + C = -4A - (7 - A) + 13A - 84 = -41$$

$$-4A - 7 + A + 13A - 84 = -41$$
$$10A = 50$$
$$A = 5$$
$$B = 2$$
$$C = 65 - 84 = \cancel{-31}\ -19$$

$$F(s) = 5 \cdot \frac{1}{s-1} + \cancel{2} \frac{2s - 19}{(s-2)^2 + 3^2}$$
$$= 5 \cdot \frac{1}{s-1} + 2 \cdot \frac{s-2}{(s-2)^2 + 3^2} - \frac{15}{3} \cdot \frac{3}{(s-2)^2 + 3^2}$$

$$f(t) = 5e^t + 2e^{2t}\cos 3t - 5e^{2t}\sin 3t$$

25. $\frac{A}{s-2} + \frac{Bs+C}{(s+1)^2+2^2} = \frac{A}{s-2} + \frac{B(s+1)+C}{(s+1)^2+2^2}$

$A[(s+1)^2+2^2] + B(s+1)(s-2) + C(s-2)$

$= 7s^2+23s+30$

$s=-1: \quad 4A-3C = 7-23+30 = 14$

$s=2: \quad 13A = 28+46+30 = 28+76 = 104$

$A = \frac{104}{13} = 8 \qquad A=8$

$4A-3C=14$

$4A-14 = 3C = 32-14 = 18 \qquad C=6$

$s=0: \quad 5A-2B-2C=30$

$5A-2C-30 = 2B = 40-12-30 = -2$

$B=-1$

$F(s) = 8\cdot\frac{1}{s-2} - \frac{s+1}{(s+1)^2+2^2} + \frac{6}{2}\cdot\frac{2}{(s+1)^2+2^2}$

$f(t) = 8e^{2t} - e^{-t}\cos 2t + 3e^{-t}\sin 2t$

(28) $s^2 F(s) + s F(s) - 6F(s) = \dfrac{s^2+4}{s^2+s}$

$$F(s)(s^2+s-6) = \frac{s^2+4}{s(s+1)}$$

$$F(s) = \frac{s^2+4}{s(s+1)(s^2+s-6)} = \frac{s^2+4}{s(s+1)(s-2)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-2} + \frac{D}{s+3}$$

$$A(s+1)(s-2)(s+3) + Bs(s-2)(s+3) + Cs(s+1)(s+3)$$
$$+ Ds(s+1)(s-2) = s^2+4$$

$\underline{s=-1}$: $0 + (-1)(-3)(2)B + 0 + 0 = 5$

$$6B = 5$$

$$B = 5/6$$

$\underline{s=0}$: $1(-2)(3)A = 4$

$$A = -2/3$$

$\underline{s=2}$: $0 + 0 + C(2)(3)(5) + 0 = 8$

$$C = \frac{4}{15}$$

$\underline{s=-3}$: $0 + 0 + 0 + D(-3)(-2)(-5) = 13$

$$D = -\frac{13}{30}$$

$$F(s) = -\frac{2}{3}\cdot\frac{1}{s} + \frac{5}{6}\cdot\frac{1}{s+1} + \frac{4}{15}\cdot\frac{1}{s-2} - \frac{13}{30}\cdot\frac{1}{s+3}$$

$$f(t) = -\frac{2}{3} + \frac{5}{6}e^{-t} + \frac{4}{15}e^{2t} - \frac{13}{30}e^{-3t}$$

(30.) $sF(s) - F(s) = \frac{2s+5}{s^2+2s+1} = \frac{2s+5}{(s+1)^2}$

$F(s)(s-1) =$

$$F(s) = \frac{2s+5}{(s-1)(s+1)^2} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$A(s+1)^2 + B(s-1)(s+1) + C(s-1) = 2s+5$

$s=-1$: $0 + 0 - 2C = -2+5$

$C = -3/2$

$s=1$: $4A + 0 + 0 = 7$ $\quad A = 7/4$

$s=0$: $A - B - C = 5$

$A - C - 5 = B = \frac{7}{4} + \frac{3}{2} - 5 = \frac{7+6-20}{4} = \frac{-7}{4}$

$B = -\frac{7}{4}$

$F(s) = \frac{7}{4} \cdot \frac{1}{s-1} - \frac{7}{4} \cdot \frac{1}{s+1} - \frac{3}{2} \cdot \frac{1}{(s+1)^2}$

$= \frac{7}{4} \mathcal{L}\{e^t\} - \frac{7}{4} \mathcal{L}\{e^{-t}\} \cancel{+ \frac{3}{2} \frac{d}{ds}\left(\frac{1}{s+1}\right)}$

$- \frac{3}{2} \mathcal{L}\{e^{-t} t\}$

$f(t) = \frac{7}{4} e^t - \frac{7}{4} e^{-t} - \frac{3}{2} t e^{-t}$